

**Neutrino Oscillations from Supersymmetry without R-parity —
Its Implications on the Flavor Structure of the Theory***

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Abstract

We discuss here some flavor structure aspects of the complete theory of supersymmetry without R-parity addressed from the perspective of fitting neutrino oscillation data based on the recent Super-Kamiokande result. The single-VEV parametrization of supersymmetry without R-parity is first reviewed, illustrating some important features not generally appreciated. For the flavor structure discussions, a naive, flavor model independent, analysis is presented, from which a few interesting things can be learned.

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We discuss here some flavor structure aspects of the complete theory of supersymmetry without R-parity addressed from the perspective of fitting neutrino oscillation data based on the recent Super-Kamiokande result. The single-VEV parametrization of supersymmetry without R-parity is first reviewed, illustrating some important features not generally appreciated. For the flavor structure discussions, a naive, flavor model independent, analysis is presented, from which a few interesting things can be learned.

1 Introduction and outline.

We discuss here a simple and specific issue — some flavor structure aspects of the complete theory of supersymmetry without R-parity addressed from the perspective of fitting neutrino oscillation data. We will first review our formulation of supersymmetry without R-parity and its application to study of neutrino masses. The formulation has been reported in Ref.[1]. It is based on a specific choice of flavor bases that allows the maximal simplification of the tree level fermion mass matrices, as well as a comprehensive treatment of all the R-parity violating (RPV) couplings together without any assumption. We will go on then to discuss a simple scenario of three neutrino masses and mixings inspired by the recent Super-Kamiokande (Super-K) result[2], incorporating it into our framework of supersymmetry without R-parity. Our concentration here is at its implication on the flavor structure of the theory. We will discuss a naive, flavor model independent, analysis from which a few interesting things can be learned. The discussion is mainly based on results presented in Ref.[3].

2 Obtaining the supersymmetrized standard model.

Let us start from the beginning and look carefully at the supersymmetrization of the standard model. In the matter field sector, all fermions and scalars have to be promoted to chiral superfields containing both parts. It is straightforward for the quark doublets and singlets, and also for the leptonic singlet. The leptonic doublets, however, has the same quantum number as the Higgs doublet that couples to the down-sector quarks. Nevertheless, one cannot simply get the Higgs, H_d , from the scalar partners of the leptonic doublets, L 's. Holomorphicity of the superpotential requires a separate superfield to contribute the Higgs coupling to the up-sector quarks. This \hat{H}_u superfield then contributes a fermionic doublet, the Higgsino, with non-trivial gauge anomaly. To cancel the latter, an extra fermionic doublet with the quantum number of H_d or L is needed. So, the result is that we need four superfields with that quantum number. As they are *a priori* indistinguishable, we label them by \hat{L}_α with the greek subscript being an (extended) flavor index going from 0 to 3.

The most general renormalizable superpotential for the supersymmetric standard model (without R-parity) can be written then as

$$W = \varepsilon_{ab} \left[\mu_\alpha \hat{L}_\alpha^a \hat{H}_u^b + h_{ik}^u \hat{Q}_i^a \hat{H}_u^b \hat{U}_k^c + \lambda'_{i\alpha k} \hat{Q}_i^a \hat{L}_\alpha^b \hat{D}_k^c + \lambda_{\alpha\beta k} \hat{L}_\alpha^a \hat{L}_\beta^b \hat{E}_k^c \right] + \lambda''_{ijk} \hat{D}_i^c \hat{D}_j^c \hat{U}_k^c, \quad (1)$$

where (a, b) are $SU(2)$ indices, (i, j, k) are the usual family (flavor) indices; λ and λ'' are antisymmetric in the first two indices as required by $SU(2)$ and $SU(3)$ product rules respectively, though only the former is shown explicitly here, $\varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, while the $SU(3)$ indices are suppressed.

At the limit where λ_{ijk} , λ'_{ijk} , λ''_{ijk} and μ_i all vanish, one recovers the expression for the R-parity preserving model, with \hat{L}_0 identified as \hat{H}_d . R-parity is exactly an *ad hoc* symmetry put in to make \hat{H}_d stand out from the other \hat{L}_i 's. It is defined in terms of baryon number, lepton number, and spin as, explicitly, $\mathcal{R} = (-1)^{3B+L+2S}$. The consequence is that the accidental symmetries of baryon number and lepton number in the standard model are preserved, at the expense of making particles and superparticles having a categorically different quantum number, R-parity. The latter is actually the most restric-

tive but not the most effective discrete symmetry to control superparticle mediated proton decay[4].

3 The single-VEV parametrization.

With the above discussion, it is clear that in the phenomenology of low energy supersymmetry, one approach worth studies is to take the complete version of a supersymmetrized standard model without extra assumption and check the phenomenological constraints on the various RPV couplings. The large number of couplings make the task sound formidable. For instance, the (color-singlet) charged fermion mass matrix is then given by

$$\mathcal{M}_c = \begin{pmatrix} M_2 & \frac{g_2 v_u}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{g_2 v_d}{\sqrt{2}} & \mu_0 & -h_{i1}^e \frac{v_i}{\sqrt{2}} & -h_{i2}^e \frac{v_i}{\sqrt{2}} & -h_{i3}^e \frac{v_i}{\sqrt{2}} \\ \frac{g_2 v_1}{\sqrt{2}} & \mu_1 & h_{11}^e \frac{v_0}{\sqrt{2}} + 2\lambda_{1i1} \frac{v_i}{\sqrt{2}} & h_{12}^e \frac{v_0}{\sqrt{2}} + 2\lambda_{1i2} \frac{v_i}{\sqrt{2}} & h_{13}^e \frac{v_0}{\sqrt{2}} + 2\lambda_{1i3} \frac{v_i}{\sqrt{2}} \\ \frac{g_2 v_2}{\sqrt{2}} & \mu_2 & h_{21}^e \frac{v_0}{\sqrt{2}} + 2\lambda_{2i1} \frac{v_i}{\sqrt{2}} & h_{22}^e \frac{v_0}{\sqrt{2}} + 2\lambda_{2i2} \frac{v_i}{\sqrt{2}} & h_{23}^e \frac{v_0}{\sqrt{2}} + 2\lambda_{2i3} \frac{v_i}{\sqrt{2}} \\ \frac{g_2 v_3}{\sqrt{2}} & \mu_3 & h_{31}^e \frac{v_0}{\sqrt{2}} + 2\lambda_{3i1} \frac{v_i}{\sqrt{2}} & h_{32}^e \frac{v_0}{\sqrt{2}} + 2\lambda_{3i2} \frac{v_i}{\sqrt{2}} & h_{33}^e \frac{v_0}{\sqrt{2}} + 2\lambda_{3i3} \frac{v_i}{\sqrt{2}} \end{pmatrix}, \quad (2)$$

from which the only definite experimental data are the three physical lepton masses as the light eigenvalues, and the overall magnitude of the electroweak symmetry breaking VEV. We must emphasize here that the easier analysis of a model with only some small number of RPV couplings admitted is, in general, lack of any theoretical motivation and of very limited experimental relevance. Even the case of having only trilinear RPV couplings in the superpotential is very difficult to motivate. Moreover, most studies of the type in the literature have extra assumptions about the scalar potential or soft supersymmetry breaking terms which are usually not explicitly addressed. This has led to quite some confusion and misleading statements in literature of the subject.

It has been pointed out in Ref.[1] that the single-VEV parametrization renders the task of studying the complete theory of supersymmetry without R-parity quite manageable. The parametrization is nothing but an optimal choice of flavor bases. In fact, doing phenomenological studies without specifying a choice of flavor bases is ambiguous. Recall that in quark physics of the standard model, there are only 10 physical parameters from the 36 real parameters of the two quark mass matrices written in a generic set of

flavor bases. To standard model physics, the 26 extra parameters are absolutely meaningless. Here for supersymmetry without R-parity, the choice of an optimal parametrization mainly concerns the 4 \hat{L}_α flavors. In the single-VEV parametrization, flavor bases are chosen such that: 1/ among \hat{L}_α 's, only \hat{L}_0 , bears a VEV; 2/ $h_{ik}^e (\equiv 2\lambda_{i0k} = -2\lambda_{0ik}) = \frac{\sqrt{2}}{v_d} \text{diag}\{m_1, m_2, m_3\}$; 3/ $h_{ik}^d (\equiv \lambda'_{i0k}) = \frac{\sqrt{2}}{v_d} \text{diag}\{m_d, m_s, m_b\}$; 4/ $h_{ik}^u = \frac{-\sqrt{2}}{v_u} V_{CKM}^\dagger \text{diag}\{m_u, m_c, m_t\}$. Under the parametrization, the (tree-level) mass matrices for *all* the fermions *do not* involve any of the trilinear RPV couplings though the approach makes *no assumption* on any RPV coupling including even those from soft supersymmetry breaking; and all the parameters used are uniquely defined. In fact, the above mass matrix is reduced to the simple form :

$$\mathcal{M}_c = \begin{pmatrix} M_2 & \frac{g_2 v_u}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{g_2 v_d}{\sqrt{2}} & \mu_0 & 0 & 0 & 0 \\ 0 & \mu_1 & m_1 & 0 & 0 \\ 0 & \mu_2 & 0 & m_2 & 0 \\ 0 & \mu_3 & 0 & 0 & m_3 \end{pmatrix}. \quad (3)$$

Each μ_i parameter here characterizes directly the RPV effect on the corresponding charged lepton ($\ell_i = e, \mu$, and τ). For any set of other parameter inputs, the m_i 's can then be determined, through a numerical procedure, to guarantee that the correct mass eigenvalues of m_e, m_μ , and m_τ are obtained — an issue first addressed and solved in Ref.[1].

4 Neutrino masses from the framework.

Under the single-VEV parametrization, the tree-level neutral fermion (neutralino-neutrino) mass matrix has also RPV contributions from the three μ_i 's only. For the discussion below, we write the mass matrix here as

$$\mathcal{M}_\mathcal{N} = \begin{pmatrix} M_1 & 0 & \frac{g_1 v_u}{2} & -\frac{g_1 v_d}{2} & 0 & 0 & 0 \\ 0 & M_2 & -\frac{g_2 v_u}{2} & \frac{g_2 v_d}{2} & 0 & 0 & 0 \\ \frac{g_1 v_u}{2} & -\frac{g_2 v_u}{2} & 0 & -\mu_0 & -\mu_1 & -\mu_2 & -\mu_3 \\ -\frac{g_1 v_d}{2} & \frac{g_2 v_d}{2} & -\mu_0 & W & 0 & Y & Z \\ 0 & 0 & -\mu_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu_2 & Y & 0 & A & C \\ 0 & 0 & -\mu_3 & Z & 0 & C & B \end{pmatrix}, \quad (4)$$

with parameters A , B , and C , and W , Y , and Z being two groups of relevant 1-loop contributions to be addressed. Setting all these to zero retrieves the tree-level result where an admixture of the three neutral fermionic states from the \hat{L}_i 's gets a nonzero mass from mixing with the gauginos and higgsinos. Note that at the limit of small μ_i 's, the neutral states correspond to ν_e , ν_μ , and ν_τ .

An important question is whether the μ_i 's are large or small. A careful analysis of an exhaustive list of constraints from tree-level leptonic phenomenology illustrates that while μ_1 has to be small, μ_2 and, especially, μ_3 do not have to [5]. In fact, MeV scale neutrino mass is easily admitted, with interesting implications on lepton number violating processes. Fitting neutrino oscillation data will then call for extensions of the model. We are interested here in the complementary scenario of sub-eV neutrino mass(es). In that case, the 1-loop contributions could also be significant. Explicitly, we assume a three neutrino scenario motivated by the recent zenith angle dependence measurement by the Super-K experiment [2]. There have been a lot of studies on the topic, details on which we are not going into here [6]. The scenario is summarized by

$$\begin{aligned}\Delta m_{\text{atm}}^2 &\simeq (0.5 - 6) \times 10^{-3} \text{ eV}^2 \\ \sin^2 2\theta_{\text{atm}} &\simeq (0.82 - 1) \\ \Delta m_{\text{sol}}^2 &\simeq (4 - 10) \times 10^{-6} \text{ eV}^2 \\ \sin^2 2\theta_{\text{sol}} &\simeq (0.12 - 1.2) \times 10^{-2}\end{aligned}$$

with $\nu_\mu - \nu_\tau$ to be responsible for the Super-K atmospheric result and MSW-oscillation of ν_e for the solar neutrino problem. The most natural setting then would be for the two neutrino mass eigenvalues of the $\nu_\mu - \nu_\tau$ system to have $m^2 \simeq \Delta m_{\text{sol}}^2$ and Δm_{atm}^2 . We will concentrate on this particular scenario below. Our concern will be focused on the compatibility of the required maximal mixing between ν_μ and ν_τ , with the general hierarchical flavor structure of the quarks and charged leptons.

Consider \mathcal{M}_ν of Eq.(4) in the $3 + 4$ block form $\begin{pmatrix} \mathcal{M} & \xi^T \\ \xi & m_\nu^0 \end{pmatrix}$. For small μ_i 's, it has a “see-saw” type structure, with the effective neutrino mass matrix given by

$$m_\nu = -\xi \mathcal{M}^{-1} \xi^T + m_\nu^0. \quad (5)$$

In the case that the μ_i contributions dominate, the first term of the equation gives

$$m_\nu = -\frac{1}{2} \frac{v^2 \cos^2 \beta (xg_2^2 + g_1^2)}{\mu_0 [2xM_2\mu_0 - (xg_2^2 + g_1^2) v^2 \sin \beta \cos \beta]} \begin{pmatrix} \mu_2^2 & \mu_2\mu_3 \\ \mu_2\mu_3 & \mu_3^2 \end{pmatrix}, \quad (6)$$

where we have neglected contributions involving the ν_e state and hence shrunk the matrix to 2×2 . Dropping the pre-factor, the matrix is diagonalized by a rotation of $\tan \theta = \mu_2/\mu_3$, giving eigenvalues 0 and $\mu_2^2 + \mu_3^2$. For this to fit in our neutrino oscillation scenario, it requires $\sqrt{\mu_2^2 + \mu_3^2} \cos \beta \sim 10^{-4} \text{ GeV}$ and $\mu_2/\mu_3 \gtrsim 0.6358$. It is interesting to note that the structure of matrix in the form $\begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix}$ naturally admits maximal mixing with a hierarchy in mass eigenvalues.

There are two types of 1-loop contributions to Eq.(4) or m_ν^0 of Eq.(5) — the quark-squark and lepton-slepton loops. The former is given by

$$(m^{LL})_{\alpha\beta}^q = \frac{3}{16\pi^2} \lambda'_{i\alpha j} \lambda'_{j\beta i} \left(\frac{A_j^d m_i^d}{\tilde{m}_{q_j}^2} + \frac{A_i^d m_j^d}{\tilde{m}_{q_i}^2} \right), \quad (7)$$

where m^{LL} corresponds to the lower 4×4 block of Eq.(4), and the soft supersymmetry breaking trilinear terms A^d are assumed to be dominantly diagonal, as generally expected. We get the dominating contribution to m_ν^0 as

$$(m_\nu^0)^q \simeq \frac{3}{8\pi^2} \frac{m_b^2}{M_{SUSY}} \begin{pmatrix} \lambda'_{33}{}^2 & \lambda'_{33} \lambda'_{33} \\ \lambda'_{33} \lambda'_{33} & \lambda'_{33}{}^2 \end{pmatrix}. \quad (8)$$

If this contribution dominates, we have a mass matrix with the same general structure as the μ_i dominating case above, hence again natural maximal mixing with a hierarchy in mass eigenvalues. It requires $\lambda' \sim 10^{-4}$ and $\lambda'_{33}/\lambda'_{33} \gtrsim 0.6358$. However, it is important to note that the natural structure would be spoiled if the μ_i contribution and the present one are at about the same level. Finally, we note also that the 4×4 form of Eq.(7) allows one to check that the contributions to the W , Y , and Z entries of Eq.(4) are really negligible.

The lepton-slepton loop contributions have a different structure. We have, similar to the the previous case,

$$(m^{LL})_{\alpha\beta}^\ell = \frac{1}{16\pi^2} \lambda_{i\alpha j} \lambda_{j\beta i} \left(\frac{A_j^\ell m_i^\ell}{\tilde{m}_{\ell_j}^2} + \frac{A_i^\ell m_j^\ell}{\tilde{m}_{\ell_i}^2} \right). \quad (9)$$

In this case, however, the antisymmetry in $\lambda \hat{L} \hat{L} \hat{E}^c$ couplings between the two \hat{L} 's gives the dominating contribution as

$$(m_\nu^0)^\ell \simeq \frac{1}{8\pi^2 M_{SUSY}} \begin{pmatrix} m_\tau^2 \lambda_{33}^2 & -m_\mu m_\tau \lambda_{32} \lambda_{33} \\ -m_\mu m_\tau \lambda_{32} \lambda_{33} & m_\mu^2 \lambda_{32}^2 \end{pmatrix}, \quad (10)$$

which is in general incompatible with large mixing. To fit in the neutrino oscillation scenario, we would hence like the lepton-slepton loops to play a secondary role, which requires λ 's of order 10^{-4} or less.

5 Flavor structure among the 4 \hat{L}_α 's

After the above discussion of the various sources of neutrino masses, let us look at the flavor structure more carefully. We will adopt a flavor model independent approach along the idea of the approximate flavor symmetry[7]. The idea is to attach a suppression factor to each chiral multiplet. For example, the down-quark mass matrix would look like

$$\mathcal{M}_d = \begin{pmatrix} \varepsilon_{Q_1} \varepsilon_{D_1^c} & \varepsilon_{Q_1} \varepsilon_{D_2^c} & \varepsilon_{Q_1} \varepsilon_{D_3^c} \\ \varepsilon_{Q_2} \varepsilon_{D_1^c} & \varepsilon_{Q_2} \varepsilon_{D_2^c} & \varepsilon_{Q_2} \varepsilon_{D_3^c} \\ \varepsilon_{Q_3} \varepsilon_{D_1^c} & \varepsilon_{Q_3} \varepsilon_{D_2^c} & \varepsilon_{Q_3} \varepsilon_{D_3^c} \end{pmatrix}$$

with the flavor hierarchy $\varepsilon_{Q_1} \ll \varepsilon_{Q_2} \ll \varepsilon_{Q_3}$ and $\varepsilon_{D_1^c} \ll \varepsilon_{D_2^c} \ll \varepsilon_{D_3^c}$. Diagonalization gives $m_d : m_s : m_b = \varepsilon_{Q_1} \varepsilon_{D_1^c} : \varepsilon_{Q_2} \varepsilon_{D_2^c} : \varepsilon_{Q_3} \varepsilon_{D_3^c}$ with mixings given by factors of the form $\varepsilon_{Q_i} / \varepsilon_{Q_j}$. We adopt the approach here for two major reasons. First of all, while an explicit flavor model may be designed to contain very specific features needed to reconcile with experimental number, the approach emphasizes on generic flavor structure features which would fit in easily any natural flavor model. If the approach can easily accommodate the required ‘‘smallness’’ of various RPV couplings, it builds a strong case for the latter couplings to be considered on the same footing as the R-parity conserving ones. Second, it is clear, from the above discussions, that we are dealing with a large number of parameters but a small amount of data. In such a situation, detailed model construction is very unlikely to be fruitful. We would rather take a humble approach and discuss issues that will not be easily washed away when more data becomes available.

In the small μ_i case considered, the \hat{L}_i basis under the single-VEV parametrization gives excellent alignment with the charged lepton mass eigenstate basis.

However, going into the approximate flavor symmetry perspective, we have to start with generic, non-diagonal, flavor bases. The mis-alignment between the two is a major problem hindering a complete discussion of the flavor structure here. This is tied up with the question of the natural values of our μ_i 's. Careful analysis of the scalar potential and vacuum solution is needed to settle the issue. We will leave that to a future studies while trying to learn something from a naive analysis.

It is easy to see that our neutrino oscillation scenario, together with the known charged lepton masses, suggests

$$\varepsilon_{L_1} \ll \varepsilon_{L_2} \sim \varepsilon_{L_3} \ll \varepsilon_{L_0} ,$$

and

$$\varepsilon_{E_1^c} \ll \varepsilon_{E_2^c} \ll \varepsilon_{E_3^c} .$$

With $\varepsilon_{L_2} \sim \varepsilon_{L_3}$, however, the factors that go with L_2 and L_3 (after a diagonalizing rotation is taken into consideration) would be $\sin 2\theta_{23} \sqrt{\varepsilon_{L_2}^2 + \varepsilon_{L_3}^2}$ and $\cos 2\theta_{23} \sqrt{\varepsilon_{L_2}^2 + \varepsilon_{L_3}^2}$. These are, of course, still of the same order of magnitude as ε_{L_2} and ε_{L_3} . If we take $\varepsilon_{E_3^c} \sim 1$, as we do with other third family factors such as ε_{Q_3} , we would use

$$\cos 2\theta_{23} \sqrt{\varepsilon_{L_2}^2 + \varepsilon_{L_3}^2} \sim \frac{m_\tau}{m_t}$$

to fix the τ mass. That is the maximal suppression in the L_3 flavor factor we can have. If we further take

$$\varepsilon_{D_3^c} \sim \frac{m_b}{m_t} ,$$

we would have naturally, at $M_{SUSY} = 100 \text{ GeV}$ [*cf.* Eqs.(8) and (10)], $\lambda'_{33} \sim \lambda'_{32} \sim 5 \times 10^{-4}$, $\lambda_{33} \sim 10^{-4}$, and $\lambda_{32} \sim 10^{-5}$. Hence, amazing enough, a bit larger value of M_{SUSY} (squark and slepton masses) would give the quark-squark loop dominating scenario naturally.

To fit our neutrino oscillation scenario with μ_i 's being the dominating neutrino mass contribution will require a higher M_{SUSY} and $\mu_1 \ll \mu_2 \sim \mu_3 \ll \mu_0$ with $\mu_3/\mu_0 < 10^{-6}$. Feasibility of this case we cannot judge, as mentioned above, until the complicated analysis of the scalar potential has been performed.

6 Summary.

In summary, from our brief analysis here, we have illustrated a few interesting issues in the flavor structure of supersymmetry without R-parity. The question of the natural suppression of the μ_i 's is important. It is however a subtle issue which has to be analyzed from a careful study of the full five-doublet ($4 \hat{L}_\alpha + \hat{H}_u$) scalar potential with the most generic soft supersymmetry breaking terms. Assuming that can be explained, we illustrate above that the suppressed values of the RPV couplings, required for fitting the limiting scenario of neutrino oscillations motivated by the recent Super-K result, fit very well into an approximate flavor symmetry perspective. Success of the latter is a strong indication that the R-parity (or lepton number) violating couplings are “naturally” small, as the light fermion masses are, and their explanation most probably lies under a common theory of flavor structure.

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